MAS 714, Fall 2019

Tutorial 2

Problem 1 Consider an array $A[1], \dots, A[n]$ that contains n distinct numbers in sorted order. Describe an algorithm that finds a position i, for which $A[i] = i$, if such a position exists. The algorithm should run in time $O(\log n)$. Prove correctness.

Problem 2 In the *Merging Problem* we are given 2 lists of n numbers each. The lists are sorted. The desired output is the sorted list of the $2n$ numbers in the lists.
a) Describe an algorithm that solves the merging problem with as few comparisons as possible.
b) We now try to prove a lower bound of $2n - o(n)$ for the number of comparisons needed. We model the problem with decision trees as follows: The decision tree has access to the result of any comparison between numbers in the first and second list, i.e., there are n^2 possible comparisons. The result is (as for sorting) the permutation that sorts the list of numbers. Show that there are $2^{2n-o(n)}$ permutations that must be distinguished by the decision tree. I.e., that the tree must have that many leaves and hence depth at least $2n - o(n)$.

Problem 3 The *diameter* of an undirected tree $V = (V, E)$ is $\max \delta(u, v)$, the largest shortest path distance among any two vertices in the tree. Describe a linear time algorithm to compute the diameter of a tree given as an adjacency list. Analyze the running time. There is no need to check that the input is really a tree. No vertex is marked as the root of the tree.

Problem 4 An undirected graph is *bipartite*, if we can partition the vertex set V into two sets U, W such that all edges are between U and V. Describe an efficient algorithm that tests if a given graph (as adjacency list) is bipartite. What is the running time?

Problem 5 The *square* of a graph $G = (V, E)$ is the graph G^2 that has the same vertices as G, and vertices u, v are connected if and only if there is a path of length exactly 2 from u to v in G. Describe an efficient algorithm that computes the adjacency matrix of G^2 from the adjacency matrix of G.