MAS 714, Fall 2019
Tutorial 4

Problem 1 A *string* is a finite sequence of characters from a finite set Γ, called the alphabet. The *edit distance* between two strings x, y is the minimum number of operations of the following 3 types we need to apply in order to change x into y.
Operation 1: Replace character x_i with another character.
Operation 2: Delete a character x_i from x.
Operation 3: Insert a character into x at position i.
Describe an efficient algorithm to compute the edit distance between x, y.

Problem 2
In this problem we consider the minimum spanning tree problem in graphs where the edges have weights that are pairwise distinct.
a) Show that there is exactly one minimum spanning tree.
b) A second-best spanning tree is a spanning tree such that the sum of edge weights is larger than for the minimum spanning tree, but smaller or equal when compared to all non-minimum spanning trees. Show that in graphs where all edge weights are pairwise distinct the second-best spanning tree is not unique in general.

Problem 3
A bottleneck spanning tree is a spanning tree T such that the maximum weight edge on T is minimized over all spanning trees in a graph G. Show how to compute a bottleneck spanning tree in time $O(m)$ for connected, undirected, weighted graphs G.

Problem 4
a) Let T and T' be minimum spanning trees of an undirected, weighted graph G. Show that the minimum edge weight in T is equal to the minimum edge weight in T'.
b) Show that for a graph G with positive edge weights, any set S of edges that connects all the vertices in G, and has minimum weight among all such sets, must form a tree.