

MAS 714, Fall 2019

Tutorial 6

Problem 1 Describe an efficient algorithm that, given an optimal solution to the dual linear program for the Max Flow problem for a graph G , capacities C and vertices s, t , finds a minimum s, t -cut.

Problem 2 Show that a Turing machine with k tapes that has time complexity at most $T(n)$ can be simulated on a Turing machine with 1 tape in time $O(kT(n)^2)$.

Problem 3 A write-once Turing machine is a single-tape Turing machine that can change each tape cell at most once. Show that any Turing machine can be simulated by a write-once Turing machine.

Problem 4 The problem LINEARINTEGERPROGRAMMING is defined like Linear Programming, except that we additionally require that an integer solution is sought, i.e., there is a real $m \times n$ matrix A and there are real vectors b and c , and we look for an integer vector x , such that $Ax \geq b$, all entries of x are nonnegative, and $c^T x$ is minimized.

A vertex cover for a graph G is a set S of vertices such that all edges of G have at least one endpoint in S .

a) Write a linear program, whose minimum *integer* solution is equal to the minimum size vertex cover.

b) Consider a (real) solution x to the linear program with minimum objective function. Show that the objective function for this solution is smaller than or equal to the size of the minimum vertex cover. Show how to derive a vertex cover from x . This vertex cover should be at most twice the size of the optimum.